Chapter 5 Review Name: ______________________

1. Evaluate $\log\left(\sqrt[6]{10}\right)$ without a calculator. Leave fractions in the form "A/B" (do not convert to decimals).

$10^x = 10^{\frac{1}{6}}$ -or- $\log_{10} 10^{\frac{1}{6}} = \frac{1}{6}$

$x = \frac{1}{6}$

2. Rewriting the statement $\log r = s$ using exponents gives

A) $10^r = s$

B) $10^s = r$

C) $10^r = 1/s$

D) $10^s = 1/r$

$10^s = r$

3. Solve $5^x = 2$ using logs. Give your answer to 3 decimal places.

$\log 5^x = \log 2$

$\frac{x \cdot \log 5}{\log 5} = \frac{\log 2}{\log 5}$

$\boxed{x = .431}$

4. Let $n = \log p$ and $m = \log q$. What is $\log(p^5q^5)$?

A) $5n + 5m$

B) $25nm$

C) $n^5 + m^5$

D) nm^{10}

$n = \log p$ $m = \log q$

$10^n = p$ $10^m = q$

$p^5 = (10^n)^5$ $q^5 = (10^m)^5$

$p^5 = 10^{5n}$ $q^5 = 10^{5m}$

$\log(p^5q^5) = \log(10^{5n} \cdot 10^{5m})$

$= \log(10^{5n+5m})$

$= \log_{10}(10^{5n+5m})$

$= 5n + 5m$

5. If the equation $Q = 8 \cdot 6^{1.3t}$ is converted to the form $Q = ae^{kt}$, then $a =$ __8__ and $k =$ __2.3293__. Give k to 4 decimal places.

$6^{1.3t} = e^{kt}$

$(6^{1.3})^t = (e^k)^t$

$6^{1.3} = e^k$

$\ln(6^{1.3}) = \ln e^k$

$1.3 \ln 6 = k$

$\boxed{k = 2.3293}$

6. Let $n = \log p$ and $m = \log q$. What is $\log \frac{p^3}{q^6}$?

A) $\frac{n^3}{m^6}$

B) $\left(\frac{n}{m}\right)^{-3}$

C) $(n-m)^{-3}$

D) $3n - 6m$

$n = \log p \quad m = \log q$

$10^n = p \quad 10^m = q$

$\log\left(\frac{(10^n)^3}{(10^m)^6}\right)$

$\log\left(\frac{10^{3n}}{10^{6m}}\right)$

$\log\left(10^{3n-6m}\right)$

$\log_{10}\left(10^{3n-6m}\right)$

$3n - 6m$

7. Let $y = 70e^{-0.01t}$, with t measured in years. What is the percent annual decay rate? Round to 3 decimal places.

$e^{-.01(1)} = .9900498 \qquad 1 - .9900498 = .00995$

$.995\%$

8. Solve for t: $\log(t - 100) = 3$

$10^3 = t - 100$

$10^3 + 100 = t$

$1000 + 100 = t$

$\boxed{1100 = t}$

9. What is the half-life of a substance that decays at a rate of 3.21% per day? Round your answer to the nearest hundredth of a day.

$\frac{1}{2}P_0 = P_0 e^{-.0321t}$

$\frac{1}{2} = e^{-.0321t}$

$\ln\frac{1}{2} = \ln e^{-.0321t}$

$\frac{\ln\frac{1}{2}}{-.0321} = \frac{-.0321t}{-.0321}$

$\boxed{21.59 \text{ days} = t}$

10. What annual interest rate, compounded monthly, is equivalent to an annual interest rate of 5.3% compounded continuously? Round your answer to the nearest thousandth of a percent, if necessary.

$\left(1+\frac{r}{12}\right)^{12} = e^{.053(1)}$

$1 + \frac{r}{12} = \left(e^{.053}\right)^{\frac{1}{12}}$

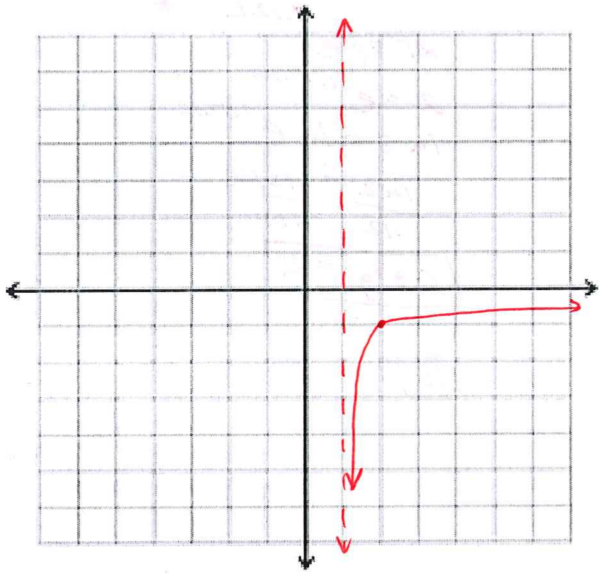
$\frac{r}{12} = \left(e^{.053}\right)^{\frac{1}{12}} - 1$

$r = 12\left(\left(e^{.053}\right)^{\frac{1}{12}} - 1\right)$

$r = .053117$

$\boxed{r = 5.312\%}$

11. Let $f(x)=\log(x-1)-1$. Graph *f*. List the asymptotes and the intercepts.



$f(2)=\log(2-1)-1$
$\log 1 - 1$
$0-1$
-1

$f(11)=\log(11-1)-1$
$\log 10 - 1$
$1-1$
0

$0=\log(x-1)-1$
$1=\log(x-1)$
$10^1=x-1$
$10=x-1$
$11=x$

$\boxed{x=1}$

$\boxed{(11,0)}$

12. Suppose the population of an endangered species is cut in half every 50 years. Assume exponential decay.
    A) What is the annual growth rate?
    B) What is the continuous growth rate?
    Give your answers correct to 4 decimal places.

A.) $\frac{1}{2}a=ab^{50}$
$\frac{1}{2}=b^{50}$
$\left(\frac{1}{2}\right)^{1/50}=b$
$.9862327$
$1-.9862327=.013767$
$\boxed{agr=-1.3767\%}$

B.) $e^{kt}=(.9862327)^t$
$(e^k)^t=.9862327^t$
$(e^k)=.9862327$
$\ln(e^k)=\ln .9862327$
$k=\ln .9862327$
$k=-.0138629$
$\boxed{GCR=-1.3863\%}$

13. If 11% of a radioactive substance decays in 6 hours, what is the half-life of the substance? Round your answer to 3 decimal places.

$.89P_0=P_0e^{k6}$
$.89=e^{6k}$
$\ln .89=\ln(e^{6k})$
$\frac{\ln .89}{6}=\frac{6k}{6}$
$-.0194223=k$

$.5=e^{-.0194223t}$
$\ln .5=\ln(e^{-.0194223t})$
$\frac{\ln .5}{-.0194223}=\frac{-.0194223t}{-.0194223}$
$\boxed{35.688\text{ h}\approx t}$

14. The half-life of a substance is 40 hours. If there are initially 100 grams of the substance, how many grams are remaining after 51 hours? Round your answer to 3 decimal places.

$\frac{1}{2}=e^{40k}$
$\ln\frac{1}{2}=\ln e^{40k}$
$\frac{\ln\frac{1}{2}}{40}=\frac{40k}{40}$

$P=100e^{-.01732867951(51)}$
$\boxed{P=41.323\text{ grams}}$

15. What is the doubling time of a population growing by 12% per year? Round your answer to the nearest hundredth of a year.

$2P = Pe^{.12t}$

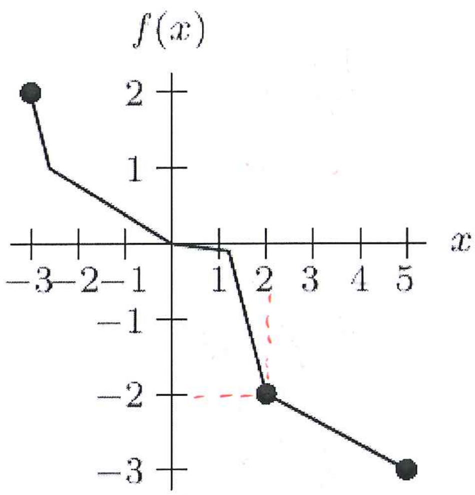
$2 = e^{.12t}$

$\ln 2 = \ln e^{.12t}$

$\frac{\ln 2}{.12} = \frac{.12t}{.12}$

$\boxed{5.78y = t}$

16. Use the following figure to evaluate $f^{-1}(-2)$. $= \boxed{2}$



17. A town with an initial population of 12,000 people doubles every 41 years. Write an equation to determine the town's population P after t years. *

$24{,}000 = 12000e^{k \cdot 41}$

$2 = e^{41k}$

$\ln 2 = \ln(e^{41k})$

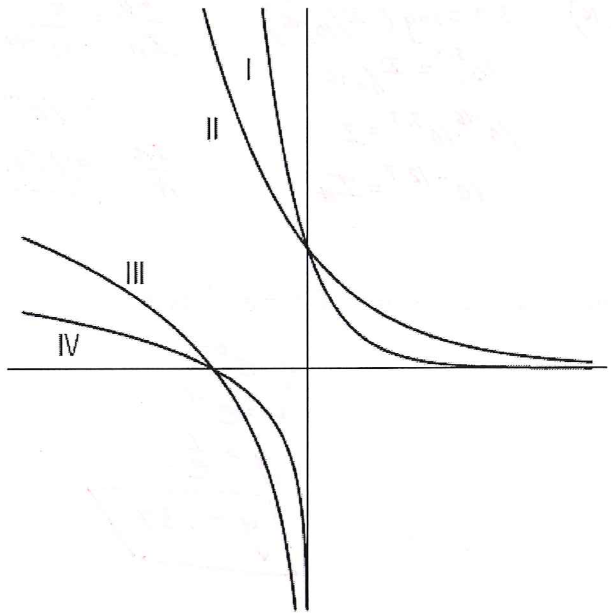
$\frac{\ln 2}{41} = \frac{41k}{41}$

$.0169060288 = k$

$\boxed{P = 12{,}000e^{.0169t}}$

18. The following figure shows the graphs of

(A). $y = e^{-x}$ (B). $y = 10^{-x}$ (C). $y = \ln(-x)$ (D). $y = \log(-x)$.



Which one is the graph of D? IV

19. The circumference, in cm, of a circle whose radius is r cm is given by $C = 2\pi r$. If $C = f(r)$, evaluate and interpret $f^{-1}(14\pi)$.

A) $28\pi^2$, the circumference of a circle with radius 14π.

B) $28\pi^2$, the radius of a circle with circumference 14π.

C) 7, the circumference of a circle with radius 14π.

D) 7, the radius of a circle with circumference 14π.

$14\pi = 2\pi r$

$7 = r$

20. Find the domain of the function $f(x) = \sqrt{-7\ln(x-6)} - 5$.

$-7\ln(x-6) \geq 0$

$\ln(x-6) \leq 0$

Graph

$\boxed{6 < x \leq 7}$

21. Sound A is 33 dB. Sound B is 55 dB. How many times more intense is Sound B than Sound A? Use dB = 10log (I/I_0) where I is the intensity of the sound and $I_0 = 10^{-16}$.

$55 = 10\log(I/10^{-16})$

$5.5 = \log(I/10^{-16})$

$10^{5.5} = \frac{I}{10^{-16}}$

$10^{5.5} \cdot 10^{-16} = I$

$10^{-10.5} = I_B$

$33 = 10\log(I/10^{-16})$

$3.3 = \log(I/10^{-16})$

$10^{3.3} = I/10^{-16}$

$10^{-16} \cdot 10^{3.3} = I$

$10^{-12.7} = I_A$

$\frac{I_B}{I_A} = \frac{10^{-10.5}}{10^{-12.7}}$

$= 10^{2.2}$

$\frac{I_B}{I_A} = \boxed{158.489 \text{ times more intense}}$

22. The vertical intercept of the graph of $y = e^{x-1}$ is at $y =$ _____. If necessary, round to 2 decimal places.

$y = e^{0-1}$

$y = e^{-1}$

$y = \frac{1}{e}$

$\boxed{y = .37}$

23. Solve for t: $200e^{0.8t} = 400e^{0.9t}$. Round your answer to 3 decimal places.

$\ln(200e^{.8t}) = \ln(400e^{.9t})$

$\ln 200 + \ln e^{.8t} = \ln 400 + \ln e^{.9t}$

$\ln 200 + .8t = \ln 400 + .9t$

$\ln 200 - \ln 400 = .1t$

$\ln\left(\frac{200}{400}\right) = .1t$

$\frac{\ln(\frac{1}{2})}{.1} = \frac{.1t}{.1}$

$\boxed{-6.931 = t}$